

Improvement of ionospheric electron density estimation with GPSMET occultations using Abel inversion and VTEC information

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[1] As it is known, the Abel transform has been proven to be a useful tool that offers the possibility to obtain vertical description of ionospheric electron density (among other neutral atmosphere parameters) through inversion of Global Positioning System (GPS) observations gathered by Low Earth Orbiters (LEO). This work is focused on extending the application of this technique to GPSMET data considering Vertical Total Electron Content. With this information, the horizontal variability of the electron concentration is accounted for. This approach shows how this information improves the results obtained in electron densities with respect to the traditional approach of Abel inversion that assumes spherical symmetry. Moreover, the ability of this inversion algorithm to obtain true-height profiles of electron density is contrasted with common procedures to obtain these profiles, especially the hmF2 parameter. *INDEX TERMS:* 2494 Ionosphere: Instruments and techniques; 2443 Ionosphere: Midlatitude ionosphere; 2415 Ionosphere: Equatorial ionosphere; 6982 Radio Science: Tomography and imaging; 6979 Radio Science: Space and satellite communication; *KEYWORDS:* Abel inversion, GPS, radio occultation, ionosphere, electron density profiles, VTEC gradients

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1. Introduction

[2] As reported in previous works, the Abel inversion applied to GPS data coming from Low Earth Orbiters (LEO) offers a valuable method to obtain vertical description of the ionospheric electron density [Hardy *et al.*, 1993; Hajj *et al.*, 1994; Hajj and Romans, 1998; Leitingner *et al.*, 1997; Schreiner *et al.*, 1999]. The classical approach assuming spherical symmetry is reported to have discrepancies with respect to ionosonde measurement of the critical frequency of the peak (i.e., foF2) from 10% to 20% (which corresponds to discrepancies of 20% to 40%, respectively, in electron density NmF2). These discrepancies depend on various factors such as the day period, latitude, and collocation between the ionosonde and the occultation. Nevertheless, the footprint of an occultation typically covers wide areas that may include high variability in the Vertical Total Electron Content (VTEC), especially in equatorial anomaly regions, leading to a mismodeling in the spherical symmetry assumption. The approach introduced by Hernandez-Pajares *et al.* [2000] overcomes this assumption by means of considering VTEC information in the inversion algorithm. A study of the discrepancies of NmE and NmF2 with the measurements of ionosondes located at midlatitude indicated the improvement of this method. This work is focused on extending this comparison considering a larger

period of 11 days and several latitudes and ionospheric conditions.

[3] The performance of Abel transform with regard to the estimation of true-height profiles of electron densities, and in particular the estimation of the electron density peak height (i.e., hmF2) value, is an issue that the same attention has not been paid as in the case of electron density. If synthetic data are used, the comparison between vertical and inverted profiles is direct. Several works have reported that the errors in the hmF2 estimation using this type of data are about 5–10 km [Hajj *et al.*, 2000]. Nevertheless, the comparison with real data is not so easy due to the fact that the number of direct measures are lower; therefore it is necessary to obtain the hmF2 using other methods for a wide comparison. The methods based on the M3000F2 parameter and the square root of the ratio between the NmF2 and NmE are easy to implement and offer an approximate value for hmF2 with an accuracy of 4% or 5% [Dudeney, 1983] for good quality ionograms. Moreover the availability of those parameters provide a high amount of values for hmF2. The Dudeney formula has been used in the work of Jakowski *et al.* [2002] for the purpose of comparison with the hmF2 value obtained with the inversion of CHAMP occultations, reporting an RMS of the discrepancies about 45 km. A more complex approach to obtain hmF2 consists of using tools such as POLAN [Titheridge, 1988] that allows to process raw ionogram data, expressed in function of virtual height, to obtain vertical profiles of electron density expressed in function

of true height, thus offering the possibility to compare the whole profile up to the peak. Those methods are considered to offer a high degree of accuracy in the true-height profiles, despite their high computational load. Depending on the quality of the ionograms, it is generally accepted that the inaccuracies of those methods can be up to 20–30 km. Regarding the intercomparison between the Dudeney formula and POLAN method, *McNamara et al.* [1987] reported a discrepancy of 10–20 km between these two techniques.

2. Abel Inversion Approach

[4] To obtain the vertical profiles of the electron density, an Abel inversion approach has been used in which information about horizontal gradients of VTEC has been added, as was done by *Hernandez-Pajares et al.* [2000]. This was done on the basis of a separability between the VTEC and a shape function ($F(h)$).

[5] As it is known, the GPS $L_I \equiv L_1 - L_2$ observable, namely the geometric free combination, can be expressed as the sum of the contribution of the electron density (N_e) in the ray path from the LEO to the GPSMET plus a bias (b_I) that contains the instrumental delays and the phase ambiguity:

$$L_I = b_I + \alpha \cdot \int_{LEO}^{GPS} N_e \cdot dl \quad (1)$$

where the α is a constant with the value $1.0506 m_{LI} \cdot m^2 / 10^{17} \text{electron}$ and the integral path is approximated by a straight line. Being λ and ϕ are longitude and latitude respectively and based on the assumption of separability, the $N_e(\lambda, \phi, h)$ can be expressed as the product of the VTEC at the desired location ($T_v(\lambda, \phi)$) and a shape function $F(h)$ (expressed in units of m^{-1}) that contains the height dependency of the profile:

$$N_e(\lambda, \phi, h) = T_v(\lambda, \phi) \cdot F(h) \quad (2)$$

[6] This expression assumes a proportionality between the T_v and N_e in the region of the occultation, following similar relations such as the slab thickness, that states the relationship between the T_v and the $NmF2$. Therefore equation (1) can be discretised and rewritten as

$$L_I(p_j) = b_I + \alpha \cdot \Delta l_p \cdot F_p \cdot T_v(\lambda_p, \phi_p) + \alpha \cdot \sum_{k=0}^j \Delta l_{jk} \cdot T_v(\lambda_k, \phi_k) \cdot F(p_k) \quad (3)$$

where Δl are the longitudes of the ray path in the corresponding layers and T_v are the known values of VTEC obtained from geographical and time interpolation of the Global Ionospheric Maps given in IONEX format computed at the Technical University of Catalonia (UPC) using ground GPS data [see *Feltens and Schaer, 1998; Hernandez-Pajares et al., 1999*]. The quantity F_p accounts for the electron density above the LEO orbit (i.e., upper ionosphere and plasmasphere) and is considered constant throughout the occultation. Besides, it is an additional unknown to estimate.

[7] In the case of spherical symmetry, the electron density profile derived from an occultation is constant in latitude/

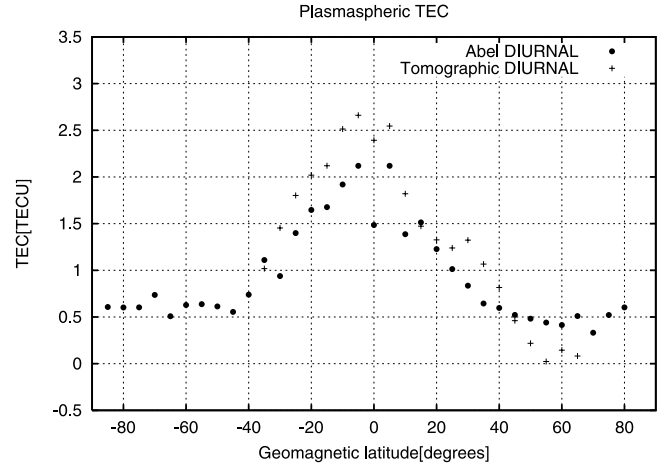


Figure 1. Estimation of upper ionospheric and plasmaspheric component obtained with the proposed method, compared with the voxel model solution. Both methods used data gathered by the GPSMET LEO satellite during year 1995, from 10 October to 21 October.

longitude. On the contrary, with this approach of separability, the shape function describes the variation in height but does not depend on geographic location. Therefore the variability in latitude/longitude is provided by the VTEC.

[8] This implementation adds two additional variables to estimate, one that accounts for the bias that affects the carrier-phase observations and the second that consists of an extra layer that takes into account the contribution of the electron density above the LEO. Since both positive and negative elevations are considered in the computation, the variation of the geometry allows the decorrelation of these two extra unknowns. Despite that the effect of the upper ionosphere and plasmasphere is not critical for LEOs such as GPSMET (whose orbit was at 750 km), it has to be taken into account if data sets gathered by LEOs at very low orbit are to be processed. This is the case of CHAMP, whose orbit is set at 400 km [*Jakowski et al., 2002*].

[9] To check the validity of the approach used to estimate the electron density content above the LEO, it has been compared with a voxel model [*Hernandez-Pajares et al., 1999*]. On the one hand, the estimation with the proposed method has been obtained processing each occultation independently using both positive and negative elevation data. On the other hand, the voxel model has been computed joining the available positive elevation data (with an elevation mask of 10°), using similar procedures to obtain VTEC estimates from ground GPS data. Figure 1 depicts the comparison between these two approaches. The values corresponding to the same geomagnetic latitude have been averaged in longitude. It can be seen how the estimation shows the expected increase of the upper ionospheric and plasmaspheric TEC in low geomagnetic latitude. A similar result is obtained during the night, where the expected decrease is stated during this period.

3. Scenario

[10] In this work the study performed by *Hernandez-Pajares et al.* [2000] has been extended to a wider scenario

of a 11 day period, from 10 October 1995 (day of year 283) to 21 October 1995 (day of year 294). To evaluate the Abel estimations of electron densities and heights, the occultations have been checked with the measured parameters given by Ionosondes. In this work, these were mainly located in Europe, North America, and Australia. Moreover, there are few comparisons for low latitudes due to the observations of the Taiwan ionosonde. This period also includes not only quiet ionospheric conditions but an ionospheric disturbance over central Europe during the day 19 October 1995 (day of year 292). The Dst indicates the presence of a disturbance (down plot of Figure 2). The upper plot of this figure corresponds to the VTEC for different ionosonde locations situated at similar longitude but different latitude. In this plot, a depletion of the VTEC in Slough and Poitiers can be seen. This effect is not present in other ionosondes such as those in the southern Europe or America.

[11] As a general procedure for parameter estimation of ionosondes, it has been considered that for a single occultation and ionosonde the valid comparisons were those made with the ionosonde measurements comprising of an interval of 1 hour centered at the epoch that the occultation took place. Besides, the maximum colocation distance between an ionosonde and the occultation was set at 2000 km.

[12] To compare the value of the ionosonde measurements, the peak of the inverted occultation has been searched. Afterward, this value is multiplied by the VTEC at the location of the ionosonde (according to equation (2)). It has to be taken into account that the separability hypothesis implies the proportional relationship between the NmF2 and TEC, in other words, a constant slab thickness during the occultation (equation (4)).

$$\tau = \frac{T_{v,NmF2}}{NmF2} \quad (4)$$

where the $T_{v,NmF2}$ and NmF2 are the VTEC and the electron density peak, respectively. Therefore the dispersion of the slab thickness in function of the local time has been studied

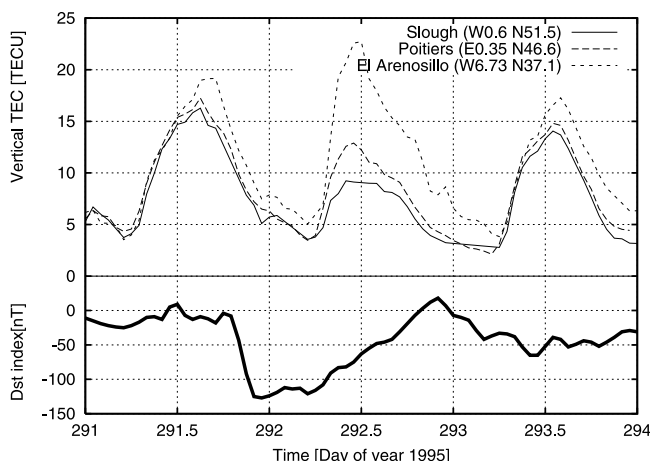


Figure 2. (top) VTEC over different locations of central Europe, (bottom) Dst index during the disturbance. Period goes from 18 October 1995 (day of year 291) to 20 October 1995 (day of year 293).

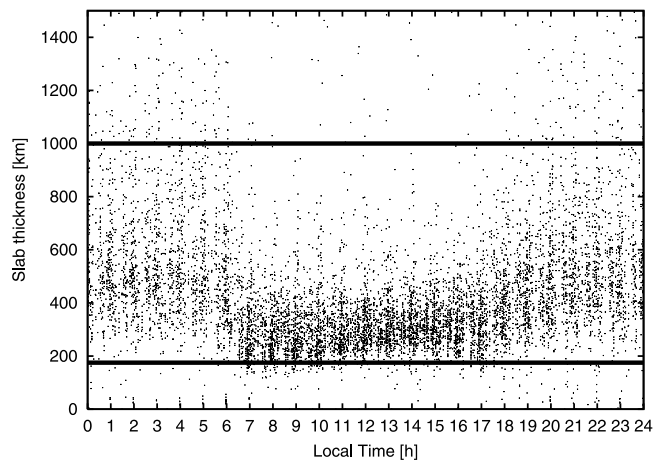


Figure 3. Dispersion plot of slab thickness in function of local time for the period from 10 October 1995 to 21 October 1995. The values of NmF2 were obtained from the 52 ionosondes available during this period. The boundaries to discard unlikely values of the slab thickness have been set at 175 km and 1000 km (also shown in the plot).

for the period considered (see Figure 3) in order to check the consistency between the measured values of peak density of the ionosondes and VTEC computed from the ground GPS data. The slab thickness values under 175 km or above 1000 km have been discarded since this may be due to a large error in the ionosonde NmF2 value and/or VTEC estimation causing a bad comparison.

[13] It can be seen that the inverted occultation can give a measure of the slab thickness by inverting the shape function at the peak of the profile ($\tau_{occultation} = 1/F(hmF2)$). Therefore the same criteria of slab thickness applied to ionosonde/TEC measurements, explained above, has been applied to each occultation as well. This filter rejects around 10% of comparisons due to unrealistic ionosonde/TEC measurements and/or bad inverted occultation. These outliers are caused mainly by ionosonde data (approximately 70% of the rejected comparisons, the remaining 30% were due to occultation data). For both the ionosonde and occultation data, the measurements with slab thickness above 1000 km were located during nighttime, coinciding with very low electron density values. On the other hand, daytime contained the majority of rejected comparisons caused by a slab thickness under 175 km.

4. Results

4.1. Frequency Estimations, foF2 and foE

[14] In the comparison stage, since ionosonde provide measurements of critical frequencies instead of electron densities, the Abel profile values of the maximum electron density at the F2 and E layers (NmF2 and NmE, respectively) have been transformed to the corresponding critical frequencies (foF2 and foE, respectively) using the well-known expression $F = 8.98 \cdot \sqrt{N_e}$, where N_e and F are expressed in e/m^3 and Hz, respectively.

[15] Table 1 summarizes the comparison of the performance between the spherical symmetry approach of Abel inversion and the one assuming separability. These results shows three cases in which the performance with respect to

Table 1. Table of foF2 Errors With Respect to Ionosonde Measurement^a

		N.comp	Sep.Hyp. RMS: MHz [%]	Sph.Symm RMS: MHz [%]
<i>Quiet Ionosphere</i>				
Low latitudes	Day	177	1.2 [16.2]	1.6 [21.4]
	D&D	10	0.5 [9.7]	0.8 [14.5]
0° ± 30°	Night	100	1.1 [27.6]	1.3 [31.7]
Middle and high latitudes	Day	2054	0.7 [11.6]	1.1 [17.8]
	D&D	908	0.8 [19.1]	1.1 [25.9]
±30° ± 90°	Night	1122	0.6 [17.6]	0.8 [23.9]
<i>Disturbed Ionosphere</i>				
Europe	Day	105	0.9 [17.7]	1.6 [31.2]
	D&D	94	0.7 [18.2]	1.2 [30.8]
	Night	32	0.4 [16.1]	0.8 [34.9]

^aThe error is Absolute RMS in MHz and Percentual relative RMS difference in brackets. The number of comparisons is also given.

the ionosonde measurements are worse: (1) Low Latitudes. Due to the Equatorial anomalies, the VTEC follows a strong nonlinear behavior. (2) Dawn and Dusk (D&D). During this period, there is a high variability in the profile (both in electron density and height). The assumption of separability is able to cope with this variability only to a limited extent. (3) Disturbed ionosphere. This condition cause high variability and nonlinear behaviors in the VTEC.

[16] Even the occurrence of the above-mentioned factors, results show how the introduction of VTEC information in the Abel inversion improves the estimations obtained assuming spherical symmetry. In fact, the average improvement is about 30%. Generally speaking, when the gradients of VTEC are not constant, it is necessary to consider VTEC data in the inversion scheme to obtain improved results with respect to the spherical symmetry assumption.

[17] Figure 4 shows a graphical interpretation of the discrepancies for two particular examples of comparison between ionosonde and inversion. The effect of the slab thickness in the comparisons is shown. Upper plot corresponds to a shape function corresponding to an occultation taking place near College (Alaska, W147.8 N64.9). In this case, the slab thickness behaves almost constantly; therefore the comparison is better than in the case in which the slab thickness is not constant. This is the case of the lower plot, where the electron density peak increases while the VTEC at the ionosonde location (Taiwan, E121.2 N25.0) decreases.

[18] Among these factors that affect the performance of the inversion, the increase of distance between the ionosonde and the occultation (i.e., lack of colocation) is an additional source of error. Figure 5 shows how the error increases as the colocation decreases (distance increases). Nevertheless, since an appropriate VTEC is considered to obtain the N(h) profile from a shape function, the error contribution of colocation is diminished when the assumption of separability is considered. In the case of the spherical symmetry, the error in the foF2 estimation increases by an average value of 0.40 MHz each time the co-location distance is increased by 1000 km. In the case of the separability assumption, this rate is reduced to 0.14 MHz/1000 km, showing more robustness of this approach with regard to the colocation error. This plot shows an average bias of -0.1 MHz in the estimation of the foF2 using the separability approach. The bias shows a certain local time

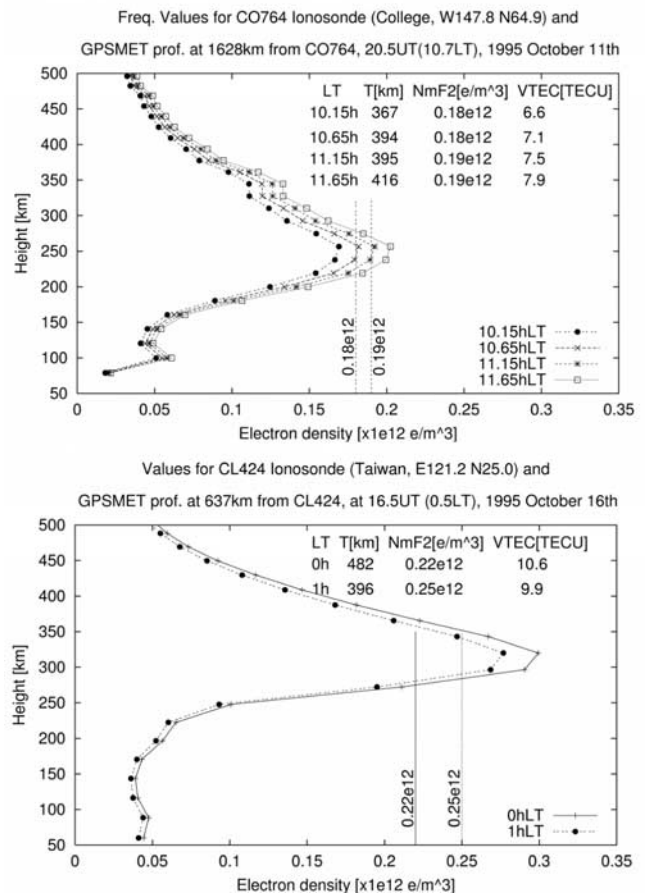


Figure 4. Effect of the variation of the slab thickness in the comparisons. The plots correspond to two different shape functions. The NmF2, VTEC and slab thickness (T) at the ionosonde location are indicated at the given epochs. To obtain the electron density profiles the shape function has been multiplied by the given VTECs.

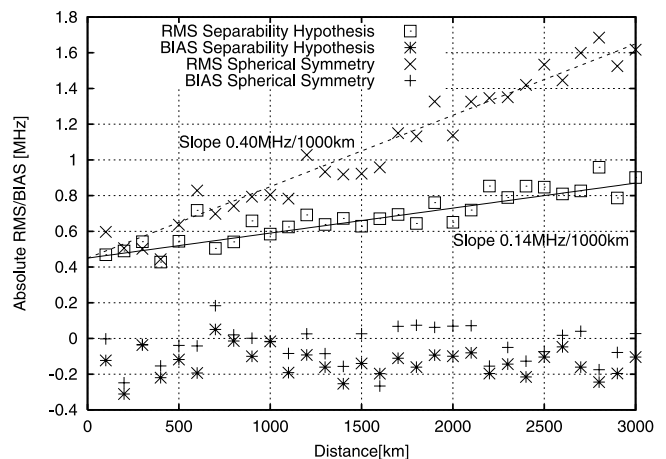


Figure 5. Effect of the colocation distance to the absolute RMS error of foF2 estimation with respect to Ionosonde measurements. Processed period of GPSMET data from 10 October 1995 to 21 October 1995. A total number of 52 ionosondes were used for comparison (mainly located at Europe and USA).

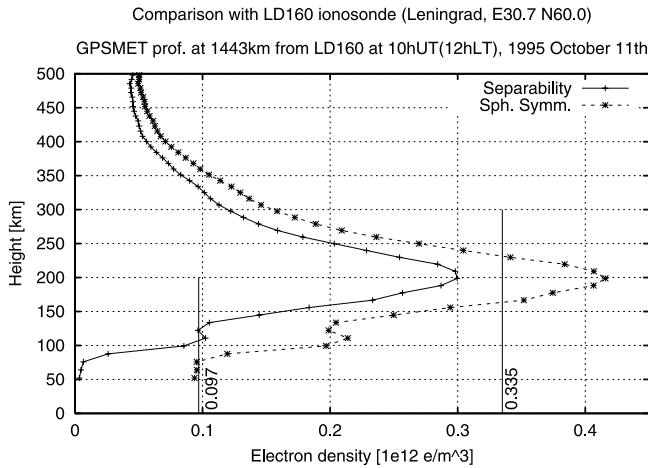


Figure 6. Effect of accumulative errors on the computation of the E layer electron density. The errors in the estimation of the upper layers strongly affects the lowermost layers. This particular example corresponds to an occultation of the GPSMET with the PRN20 at 11 October 1995 (day of year 284), 1000 UT (1200 LT), at E43.3 N44.9, compared with the corresponding NmF2 and NmE values obtained with the Leningrad Ionosonde.

dependency in both the spherical symmetry and separability hypothesis approach, this reported negative bias corresponds to the daytime period, which contains the larger number of comparisons (nearly 30% of all comparisons correspond to the time period from 9 hLT to 11 hLT). The bias is due to a mismodeling caused by the assumption of a shape function as a profile descriptor. Ideally, the integral of this shape function should be 1; nevertheless, actual inversions offer discrepancy with respect to this value. The typical values of this integral for the occultations processed is around 0.8 which explains this negative bias.

[19] Regarding the estimation of the foE the comparative performance of the two Abel approaches has been studied as well. Since the Abel inversion is a recursive method in which the upper layers are solved before the lowermost ones, the error in the estimation of the former affects dramatically the later. Although the importance of this effect may depend on the occultation, the assumption of the separability helps to diminish this effect as shown in Figure 6 and Table 2. To compare the two approaches for the Abel inversion, the occultations with the presence of a peak in the interval 90–130 km have been searched for.

[20] Again, the values of the shape function are multiplied by the corresponding VTEC at the location of the ionosonde. Afterwards, it has been compared with the values of foE and foEs provided by the ionosonde. The discrep-

Table 2. Table of foE Errors With Respect to Ionosonde Value^a

	N.comp	Sep. Hyp. RMS: MHz [%]	Sph. Symm. RMS: MHz [%]
E layer	135	0.4 [17.1]	0.7 [28.5]
Es layer	35	0.5 [16.2]	1.0 [30.4]

^aThe error is Absolute RMS in MHz and Percentual relative RMS difference in brackets.

Table 3. Intercomparison Between the POLAN, Dudeney Formula and Abel Inversion for hmF2 Estimation^a

	POLAN-DUD			Abel-POLAN			
	N.comp	bias, km	σ , km	N.comp	bias, km	σ , km	Abel-DUD bias, km
$foF2/foE \leq 2$	39	15.9	20.8	65	-12.6	27.0	4.1
$foF2/foE > 2$	174	0.5	9.3	252	-0.8	22.1	0.8

^aOnly the common comparisons between methods have been considered. The values for the bias and standard deviation are expressed in kilometers.

ancies are summarized in Table 2. As in the case of foF2, the results considering separability hypothesis improve by an average figure of 40% those obtained with spherical symmetry.

4.2. The hmF2 Estimation

[21] The GPSMET data used in this work were sampled at every 10 s. This implies that the height resolution is of few kilometers in the upper layers and it decreases for the lower layers reaching to the impact parameter separation of 20 km near the peak of the F2 layer and below. Therefore the expected discrepancies with a reference value of hmF2 has to be at least 20 km. As shown in the tables of this section (Table 3 and Table 4), the discrepancies are slightly greater. Regarding the discrepancies in the reference values of hmF2 using separability hypothesis and spherical symmetry, these are small enough to state that those methods offer the same performance with regard to height estimation.

[22] When the hmF2 estimation is performed with synthetic data and considering separability hypothesis, the discrepancies are compatible with the 5–10 km reported by *Hajj et al.* [2000]. In order to use real data instead, two methods have been considered to obtain the hmF2 estimates: the POLAN true-height inversion method and the Dudeney formula based on the M3000F2 parameter. In the case of the Dudeney formula, only the values that verified the condition that $foF2/foE > 1.215$ have been considered as done by *McNamara et al.* [1987]. The POLAN profiles for the ionosondes of Lerwick, Chilton and Port Stanley have been obtained from the World Data Center-A in RAL (available at <http://www.wdc.rl.ac.uk>). The expected error for the estimation of the hmF2 using POLAN reported for this data set is 3 km approximately. Despite the fact that the comparative study of the hmF2 values obtained by both methods is beyond the scope of this work, Table 3 shows a comparison between these methods and the hmF2 value obtained from GPSMET occultations. The ionosondes considered for this table correspond to the ones that POLAN profiles were available. *McNamara et al.* [1987] reported a bias of the Dudeney formula with respect to the POLAN method. Moreover, it stated that the bias is dependent on the ratio between the peak frequencies of the F2 and E layers ($foF2/foE$). Following this criteria, the height comparisons

Table 4. Comparison With Dudeney Formula for All Available Occultations^a

	N.comp	bias, km	σ , km
Abel-DUD			
$foF2/foE \leq 2$	682	-2.7	24.0
$foF2/foE > 2$	1775	-10.8	25.8

^aThe values for the bias and standard deviation are expressed in kilometers.

obtained in this work have been divided according to this ratio. It can be seen that the bias follow the same behavior (the larger the frequency ratio the lower the bias). Regarding the standard deviation in the comparison of Abel with the other models, it can be seen that it is similar in both cases, close to 25 km. To give particular examples of the performance in the true-height profiling compared with a standard method of inverting ionograms such as POLAN, Figure 7 shows different inverted occultations compared with the corresponding profiles obtained with the POLAN method.

[23] A summary of the Abel inversion performance as an hmF2 estimator (compared with the Dudeney formula, using all available ionosonde data) is showed in Table 4. These results indicate that the dependency of the bias with the ratio $foF2/foE$ has a special significance in the RMS comparison.

[24] Figure 8 shows the bias and standard deviation for several ionosondes. It can be seen that the bias depends on the ionosonde; as a consequence, the overall standard deviation will suffer a certain increase. The standard deviation is dependent on the ionosonde, but a slight increasing as latitude decreases can be seen. This could be explained by the fact that the assumption of the separability for the lower latitudes is valid to a certain extent (i.e., for these

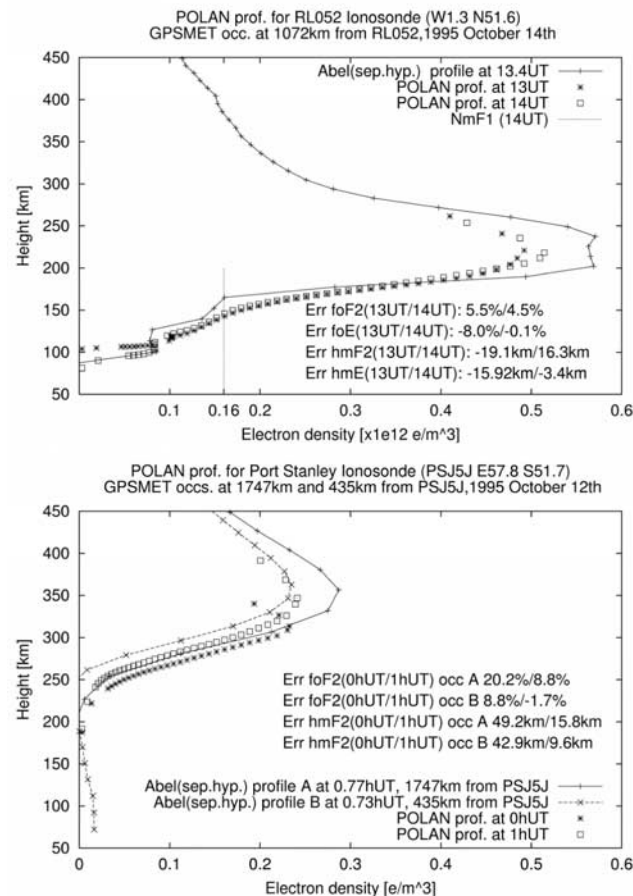


Figure 7. Inverted profiles using GPSMET data and separability hypothesis. The comparison is made with POLAN profiles obtained from ionogram data.

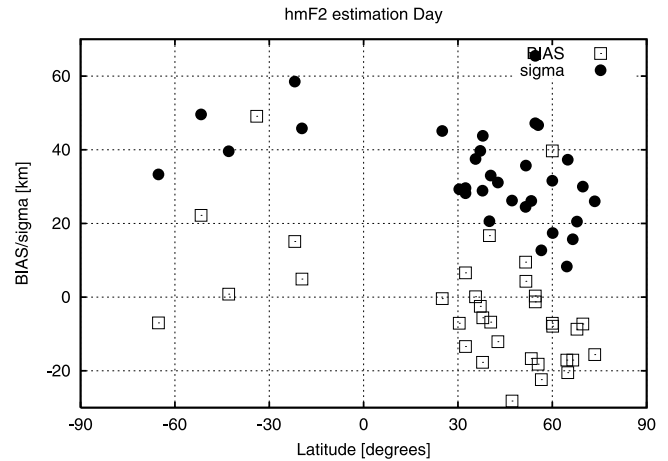


Figure 8. Latitude variation of the standard deviation. The bias and standard deviation are plotted for each ionosonde. The reference values for the hmF2, due to its wider availability, have been obtained from the Dudeney formula applied to the period of 10 October 1995 to 21 October 1995. There were 32 ionosondes for which the parameters foF2, foE, and M3000F2 were simultaneously available.

latitudes, only one shape function can not fully describe the height variation of the profile).

5. Discussion and Conclusions

[25] This work showed how considering VTEC information in the Abel transform improves the estimations of foF2 and foE by an average value of 30% and 40%, respectively, obtained assuming spherical symmetry, confirming the results obtained previously. This improvement is verified in all latitudes, for all local times and for different ionospheric conditions. Moreover, the assumption of the separability between the VTEC and the shape function causes a diminishing effect in the error caused by the collocation distance between the occultation and the reference value of the ionosonde.

[26] Nevertheless, regarding the true-height estimation of the profile, the assumption of the separability offers similar results as in the case of spherical symmetry. It has been showed that the expected performance of Abel inversion regarding the hmF2 estimates is near 25 km of standard deviation (with a slight dependency on latitude). The bias contribution of the discrepancy with the Dudeney formula depends on the ratio between the frequency peaks of the F2 and E layers. To sum up, taking advantage of the shape functions and VTEC information, it is possible to obtain, in an easy way, a wider description of the true-height profiles of the ionospheric electron density.

[27] **Acknowledgments.** We are grateful to the UCAR for the GPSMET data. The ionosonde values for the frequencies and M3000F2 are taken from the SPIDR web server (available at <http://spidr.ngdc.noaa.gov>) and the POLAN inverted profiles from the World Data Server at the Rutherford Appleton Laboratory (available at <http://www.wdc.rl.ac.uk>). We are grateful to Yuei-An Liou of National Central University of Taiwan for the GPS and ionosonde data over Taiwan. This work has been partially supported by the “Generalitat de Catalunya” under the fellowship 2000FI-00395 and the Spanish projects TIC-2000-0104-P4-03 and TIC-2001-2356-C02-02.

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