
Magnitudes and Moments of Earthquakes

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1. EARTHQUAKE MAGNITUDE

Charles Richter's [29] original magnitude scale, called the local magnitude M_L , was designed to provide a relative measure of the size of waves recorded in southern California from local earthquakes, i.e.,

$$M_L = \log A + B \quad (1)$$

where A is the maximum trace amplitude (in mm) on records of specific Wood-Anderson torsion seismographs and B is the distance correction. The predominant period of the waves used is usually 0.1-3 s. To generalize the restrictive range of earthquakes that could be quantified with M_L , Gutenberg [17] used the amplitude of distant surface waves in his definition of the surface-wave magnitude M_s , and he thus extended the Richter's scale to observations at any distance. M_s is defined as

$$M_s = \log A + 1.656 \log D + 1.818 \quad (2)$$

where A is the maximum combined horizontal ground amplitude (in microns) for surface waves with periods of about 20 s and D is the angular epicentral distance (in degrees). Gutenberg [18] and Gutenberg and Richter [20] also used the maximum amplitude of body waves to define the body-wave magnitude, m_B :

$$m_B = \log (A/T) + Q \quad (3)$$

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where A is the maximum ground amplitude (in microns), T is the corresponding period (in s), and Q is the correction function for the type of body-wave phase (P, PP and S), epicentral distance and focal depth. Usually the period of the waves used ranges from 4-20 s [8].

Since these formulas were established, many variations of these scales have been proposed for different regions, magnitude ranges and instruments by many investigators. Several tens of the existing relations are compiled by Bath [12]. These scales have not always been calibrated well against the original scale, and mixed use of inhomogeneous magnitude values requires some caution [3, 25]. There is significant inhomogeneity in the existing magnitude catalogs [16, 3]. For example, the single magnitude values in the famous catalog of Gutenberg and Richter [19] are basically M_s for large shocks shallower than 40 km, but are m_B or modified m_B for large shocks deeper than 40 km [3, 5].

The body-wave magnitude of current use, denoted by m_b , is determined routinely from the amplitude in the first few cycles of 1-s period P waves on "narrow-band" instruments. Owing to the substantial difference in the wave period and the way the maximum amplitude is measured, m_b and m_B are distinct and should not be used as the same scale.

Extensive studies by Gutenberg and Richter suggested that M_s , m_B and M_L are related to each other and can be used to represent the total energy (E) released in seismic waves. Through repeated revisions, Gutenberg and Richter [20] derived empirical relations:

$$m_B = 1.7 + 0.8M_L - 0.01M_L^2 \quad (4)$$

$$m_B = 0.63M_s + 2.5 \quad (5)$$

$$\text{or equivalently } M_s = 1.59m_B - 3.97 \quad (5)$$

$$\log E(\text{J}) = 2.9 + 1.9M_L - 0.024M_L^2 \quad (6)$$

$$\log E(\text{J}) = 1.5M_s + 4.8 \quad (7)$$

The quantification of earthquakes has been a problem with a very complex history, because of difficulty in representing earthquake size by a single parameter. Recent progress in seismology provided more quantitative scales than the magnitude. According to elastic dislocation theory, an earthquake can be modeled by shear faulting. In this model, energy and potency are physically meaningful parameterizations of earthquake size even in a

earthquakes have been available on a routine basis in *Monthly Listing of Preliminary Determination of Epicenters* by U.S. Geological Survey since July, 1981.

Table 2 lists a selection of the values of surface-wave magnitude M_s , seismic moment M_0 , fault length L , fault width W and average dislocation on the fault D for shallow earthquakes. The selection illustrates variation of source parameters over a wide range of the magnitude. For an

